Viewing the world systemically.

ATIS: Initial Axiom Set

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ATIS Report

The SimEd Basic Logic as Founded on the Logic of Axiomatic Theory of Intentional Systems:

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ATIS Initial Axiom Set

Presented in this report are the axioms that have been selected for an initial education theory. Of course, with 201 axioms from the SIGGS Theory, many more axioms could have been chosen, and will need to be chosen for any in-depth analysis of an educational system or any other type of system. Only 17 axioms have been chosen here, as described below, so that the analysis does not become too complex.

These axioms were selected from the list of 201 SIGGS Hypotheses. The SIGGS Hypotheses were in fact developed as "axioms," however were referred to as "hypotheses" so as to be consistent with the prevailing nomenclature used in education. However, for *ATIS*, the more accurate representation of "axiom" will be used.

Initially 57 SIGGS Hypotheses were selected for consideration. Upon review, several of these were eliminated due to the nature of their construction. That is, initially no axiom will be used that references any minimum or maximum value or a time reference.

After deleting the initially unacceptable SIGGS Hypotheses, 39 were left. Upon further review of these 39, it was found that 14 of these SIGGS Hypotheses were in fact theorems.

The identification of these theorems demonstrates the value of a formal theory that is rigorously developed using logico-mathematical tools. While SIGGS did present a formal statement of the properties and hypotheses, there was not time to do the extensive work required to assure consistency of the formalizations and to present them in a form that would allow for the type of analysis that would reveal that some of the hypotheses are in fact theorems. *ATIS* has been developed to resolve this problem by presenting a consistent and rigorous definition of the properties. The value of this development is now recognized by its use in analyzing the SIGGS Hypotheses.

As a result of the analysis of the SIGGS Hypotheses, 14 theorems have been identified. To maintain a correlation with SIGGS, they have been identified by the designation "T.x" ("Theorem.x") where "x" is the SIGGS Hypothesis number. These theorems are presented following the presentation of the 17 initial axioms.

Another problem that is addressed in this report results from the concerns of Theodore W. Frick who has strongly emphasized the problems of logical analysis that arise when a temporal factor is involved. The concern involves the distinction between the traditional logic of 'implication' as opposed to the logic of 'causation'. The distinction involves the temporal factor of 'causation' that does not occur in logical 'implication'. This distinction can now be addressed in *ATIS*.

For *ATIS* logic, the following relations are introduced.

'Temporal implication' or 'causation' will be defined as a premise that temporally precedes in time the conclusion, and will be designated by '⊐'. The choice of symbol is consistent with the symbol, '⊃', used for logical implication. The logical distinction between the uses of the symbols is determined by the rules for their use. Essentially, they intuitively can be interpreted as traditionally read; that is, "If …, then …", or, in the case of '⊐', it can be read as "… causes …". Preferably, however, it is recommended that both be read as "… implies …", and the temporal designation is determined by the symbol used. In addition to '⊐', the symbol '⊒' for "precedes or equals in time", and the symbol'⋥' for "strictly precedes in time" may also be used where greater precision is required.

Rules for '⊃' and '⊐'.

(1) Unless a temporal factor is required, the traditional logical implication, '⊃', will be used.

(2) Where causation or a clear need is indicated for the use of a temporal relation is required, then '⊐' will be used.

(3) Where '⊃' is used, then all rules of the *Sentential* and *Predicate Calculi* will apply.

(4) Where '⊐' is used, no negation-equivalence rules apply. For example, 'P ⊐ Q' is not equivalent to '~Q ⊐ ~P', as it would be for '⊃'.

(5) The only rule of inference for '⊐' is Modus Ponens.

As a guide to determine which "implication" to use, any statement that designates a property as "increasing," "decreasing," or designating a time component will require the use of '⊐' rather than the standard logical notation of '⊃'. As will be seen, all of the axioms in the Initial Axiom Set have been converted so as to use '⊐' instead of '⊃'. This change will greatly impact the type of theorems that can be generated as a result of the limitation on the proofs.

Initial Axiom Set

[The numbers of the axioms refer to their listing in *Steiner and Maccia*.[1]]

A.10. System input decreases, implies (input-dependent) fromput decreases.

A.15. System output increases, implies fromput increases.

A.28. System filtration increases, implies adaptability increases.

A.33. System toput increases and fromput increases, implies feedthrough increases.

A.35. System input is constant and fromput is constant, implies output is constant.

A.90. System toput increases, implies centrality decreases.

A.91. System feedin decreases, implies unilateralness decreases.

A.93. System feedin decreases, implies complexity-degeneration increases.

A.100. System complete-connectivity increases, implies feedin increases.

A.102. System interdependence increases, implies feedin increases.

A.105. System centrality increases, implies toput decreases.

A.106. System complete-connectivity increases or strongness increases, implies toput increases.

A.107. System complete-connectivity increases or strongness increases, implies input increases.

A.144. System filtration decreases, implies isomorphism increases.

A.151. System isomorphism increases; implies fromput decreases, and feedout decreases.

A.194. System size increases, and complexity-growth is constant; implies toput increases.

A.195. System size increases, and complexity-growth is constant; implies feedin decreases.

[1] Maccia, Elizabeth Steiner and George S. Maccia (1966), *Development of Educational Theory Derived from Three Educational Theory Models*, The Ohio State University, Research Foundation, Columbus, Ohio.

Theorems Derived from the Initial Axiom Set

[The numbers of the theorems refer to their hypothesis listing in *Steiner and Maccia*.]

T.12. System input increases, implies filtration decreases.

T.13. System input decreases, implies filtration increases.

T.21. System feedthrough increases, implies compatibility increases.

T.29. System openness increases, implies efficiency decreases.

T.53. System complete-connectivity increases, implies flexibility increases.

T.54. System strongness decreases, implies wholeness increases.

T.55. System strongness increases, implies hierarchical-order decreases.

T.56. System strongness increases, implies flexibility increases.

T.57. System unilateralness, implies hierarchical-order.

T.179. System size increases, and complexity-growth is constant; implies vulnerability increases.

T.180. System size increases, and complexity-growth is constant; implies flexibility decreases.

T.181. System size increases, and complexity-growth is constant; implies centrality decreases.

T.182. System size is constant, and complexity-degeneration increases; implies disconnectivity increases.

T.183. System size decreases, and complexity-degeneration increases; implies disconnectivity decreases.

Formalization of *ATIS* Axioms

A.10. $I_{\mathcal{P}}^{\downarrow} \sqsupset F_{\mathcal{P}}^{\downarrow}$

A.15. $O_{\mathcal{P}}^{\uparrow} \sqsupset F_{\mathcal{P}}^{\uparrow}$

A.28. ${}_{\mathbb{S}}\mathscr{F}^{\uparrow} \sqsupset {}_{A}\mathbb{S}^{\uparrow}$

A.33. $T_{\mathcal{P}}^{\uparrow} \wedge F_{\mathcal{P}}^{\uparrow} \sqsupset \oint_{T}^{\uparrow}$

A.35. $I_{\mathcal{P}}^{c} \wedge F_{\mathcal{P}}^{c} \sqsupset O_{\mathcal{P}}^{c}$

A.90. $T_{\mathcal{P}}^{\uparrow} \sqsupset {}_{C}\mathscr{O}^{\downarrow}$

A.91. $\oint_{I}^{\downarrow} \sqsupset {}_{U}\mathscr{O}^{\downarrow}$

A.93. $\oint_{I}^{\downarrow} \sqsupset \mathcal{X}^{-\uparrow}$

A.100. ${}_{CC}\mathscr{O}^{\uparrow} \sqsupset \oint_{I}^{\uparrow}$

A.102. ${}_{I}\mathscr{O}^{\uparrow} \sqsupset \oint_{I}^{\uparrow}$

A.105. ${}_{C}\mathscr{O}^{\uparrow} \sqsupset T_{\mathcal{P}}^{\downarrow}$

A.106. ${}_{CC}\mathscr{O}^{\uparrow} \vee {}_{S}\mathscr{O}^{\uparrow} \sqsupset T_{\mathcal{P}}^{\uparrow}$

A.107. ${}_{CC}\mathscr{O}^{\uparrow} \vee {}_{S}\mathscr{O}^{\uparrow} \sqsupset I_{\mathcal{P}}^{\uparrow}$

A.144. ${}_{\mathbb{S}}\mathscr{F}^{\downarrow} \sqsupset \mathscr{I}^{\uparrow}$

A.151. $\mathscr{I}^{\uparrow} \sqsupset F_{\mathcal{P}}^{\downarrow} \wedge \oint_{O}^{\downarrow}$

A.194. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \sqsupset T_{\mathcal{P}}^{\uparrow}$

A.195. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \sqsupset \oint_{I}^{\downarrow}$

Formalization of *ATIS* Theorems

T.12. $\mathrm{I}_{\mathcal{P}}^{\uparrow} \sqsupset {}_{\mathbb{S}}\mathscr{F}^{\downarrow}$

T.13. $\mathrm{I}_{\mathcal{P}}^{\downarrow} \sqsupset {}_{\mathbb{S}}\mathscr{F}^{\uparrow}$

T.21. $\oint_{\mathrm{T}}^{\uparrow} \sqsupset \mathsf{C}^{\uparrow}$

T.29. ${}_{\mathrm{O}}\mathbb{S}^{\uparrow} \sqsupset {}_{\mathrm{EF}}\boldsymbol{\mathcal{S}}^{\downarrow}$

T.53. ${}_{\mathrm{CC}}\mathscr{C}^{\uparrow} \sqsupset {}_{\mathrm{F}}\mathscr{C}^{\uparrow}$

T.54. ${}_{\mathrm{S}}\mathscr{C}^{\downarrow} \sqsupset \mathcal{W}^{\uparrow}$

T.55. ${}_{\mathrm{S}}\mathscr{C}^{\uparrow} \sqsupset {}_{\mathrm{HO}}\mathscr{C}^{\downarrow}$

T.56. ${}_{\mathrm{S}}\mathscr{C}^{\uparrow} \sqsupset {}_{\mathrm{F}}\mathscr{C}^{\uparrow}$

T.57. ${}_{\mathrm{U}}\mathscr{C} \supset {}_{\mathrm{HO}}\mathscr{C}$

T.179. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{\mathrm{c}} \sqsupset {}_{\mathrm{V}}\mathscr{C}^{\uparrow}$

T.180. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{\mathrm{c}} \sqsupset {}_{\mathrm{F}}\mathscr{C}^{\downarrow}$

T.181. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{\mathrm{c}} \sqsupset {}_{\mathrm{C}}\mathscr{C}^{\downarrow}$

T.182. $\mathcal{Z}_{\mathrm{c}} \wedge \mathcal{X}^{-\uparrow} \sqsupset {}_{\mathrm{D}}\mathscr{C}^{\uparrow}$

T.183. $\mathcal{Z}^{\downarrow} \wedge \mathcal{X}^{-\uparrow} \sqsupset {}_{\mathrm{D}}\mathscr{C}^{\downarrow}$

It is noted that since T.57 has no temporal component, '$\supset$' is used instead of '$\sqsupset$'.

Since these theorems have been proven with *ATIS*, empirical testing can now validate them. Since all of these theorems can be proved from property definitions, this validation will help to establish the validity of the defined properties. If any theorem is found to be invalid, then it will signal a possible review of the formalization of the relevant properties.

Since *ATIS* has been designed to address questions relating to education-type or terrorist-type systems, any validation must be performed with systems of those type.

A proof of T.53 is provided on the following page to demonstrate how these proofs are carried out. The generalization of this theorem to n components should be obvious.

T.53. $\vdash {}_{CC}\mathcal{O}^{\uparrow} \supset {}_{F}\mathcal{O}^{\uparrow}$

Proof:

Let $x,y,z \in {}_{CC}\mathcal{O}$	Assumption
$\therefore\ (x,y),(x,z),(y,x),(z,x),(y,z),(z,y) \in {}_{cc}\boldsymbol{E}$	Def of ${}_{CC}\mathcal{O}$
$(x,y),(x,z),(y,x),(z,x),(y,z),(z,y) \in {}_{cc}\boldsymbol{E} \supset$ $(x,y),(x,z),(y,x),(z,x),(y,z),(z,y) \in {}_{pc}\boldsymbol{E}$	Def of ${}_{cc}E$

Therefore, x is connected to z through y and directly.

${}_{CC}\mathcal{O}^{\uparrow}$	Assumption
Let $(x,w) \in {}_{cc}\boldsymbol{E}$ be a new path connection	From Previous Step
$\therefore\ (x,y),(x,z),(y,z),(z,y),(x,w),(w,x),(y,w),(w,y),(z,w),(w,z) \in {}_{pc}\boldsymbol{E}$	Def of ${}_{cc}E$
$\therefore\ {}_{F}\mathcal{O}^{\uparrow}$	Def of ${}_{F}\mathcal{O}$ and $^{\uparrow}$
$\therefore \vdash {}_{CC}\mathcal{O}^{\uparrow} \supset {}_{F}\mathcal{O}^{\uparrow}$	Q.E.D.

****Completely-connected components**, ${}_{CC}\mathcal{O}$, $=_{df}$ a set of system components that are pair-wise path-connected in both directions.

$${}_{CC}\mathcal{O} =_{df} \mathfrak{X} = \{x \mid x \in \mathfrak{S}_0 \wedge \exists y[(x,y) \in {}_{c}\boldsymbol{E} \supset (x,y) \in {}_{cc}\boldsymbol{E}]\}$$

Completely connected is defined as a set of components of the object set; such that, if the components are connected, then they are completely connected.

****Flexible-connected components,** ${}_{F}\mathcal{O}$, $=_{Df}$ subgroups of system components that are independently path-connected between two other components not in the subgroups.

$${}_{F}\mathcal{O} =_{df} \mathfrak{X} = \{x \mid x \in \mathfrak{S}_0 \wedge \exists y[(x,y) \in {}_{c}\boldsymbol{E} \supset (x,y) \in {}_{pc}\boldsymbol{E} \wedge \mathcal{F}(\mathfrak{X}_i)((x,y))]\};\text{ where}$$

$$\mathcal{F}(\mathfrak{X}_i) = \{\mathfrak{X}_i \mid \mathfrak{X}_i \subset \mathfrak{S}_0 \wedge i>1 \wedge \forall \mathfrak{X}_i\ \exists x \in \mathfrak{S}_0\ \exists y \in \mathfrak{S}_0[(x,y) \in {}_{pc}\boldsymbol{E} \supset (x,\mathfrak{X}_i),(\mathfrak{X}_i,y) \in {}_{pc}\boldsymbol{E}]\}$$

Flexible-connected components is defined as a set of components of the object-set; such that, the components are path-connected and are path-connected through two or more subgroups of the object-set.