Viewing the world systemically.

ATIS: Structural Axioms

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The Value of Topological and Logical Analyses For Behavioral Theory Development

The value of theory in general, and behavioral or education theory in particular, is that theory provides a means of **predicting outcomes**. To date, behavioral and education sciences have had to rely upon empirical testing to arrive at predictive assertions. That is, given a hypothesis, experiments must be conducted in order to *validate* the hypothesis. Since our main concern is with education research, such research will be referenced in what follows with the understanding that the same could be said of behavioral research in general.

There are several problems with hypothesis-driven research. The first, and probably the greatest hindrance to developing a consistent and comprehensive education theory, is the belief that simply combining numerous validated hypotheses can develop a theory. Hypotheses, by their very construction, cannot specify all of the underlying assumptions upon which the hypotheses are based, let alone controlling the experiments. Combining what are considered verified hypotheses does not result in theory.[1]

The second difficulty with this type of testing and any conclusions derived therefrom is that they are dependent upon statistical measures that are only ***group-predictive***, and not ***individually predictive***. Statistical-based analyses can never be individually predictive. Hence, the value to education theorizing and its applications to individual systems, whether students or school systems, is highly questionable.

However, by analyzing the *structure* of an education system (whether the school system or any part or individual in that system), conclusions; that is, predictions can be obtained from a ***parametric analysis*** of the system. An added value to this type of analysis is that predictions relating to education systems can be made from their ***nonempirical structural parameters***. In fact, this is the only feasible way to ever analyze an education system with any assurance of the reliability of any outcomes. The reason is due to the very large number of structures contained in even the smallest education system. *ATIS* generates **thousands** of theorems which, when applied to specific education systems, will result in **millions** of possible *hypotheses* (that is, *theorems* or *empirical predictions*) being generated. Nothing could ever be achieved if we had to wait for verification of every hypothesis that can be obtained.

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[1] See *ATIS Theory Development / Logic, Models and Theories / Types of Systems / Research Methodologies*. For this report, go to: https://www.researchgate.net/profile/Kenneth_Thompson7/publications

Scope of Initial Research

Since our concern here is with the *structural properties*, only the axioms from SIGGS that contain these properties will here be considered.

Following these axioms is a logico-mathematical formalization of the axioms.

Following this formalization, theorems of the theory are presented. These theorems will be presented in two ways:

(1) Some theorems will be explicitly stated; and

(2) Theorem schemas will be given indicating what theorems can be made explicit.

This second approach is taken due to the large number of theorems that this theory produces. For example, one theorem schema, *Logical Schema 2*, will produce 3,240 theorems from the axioms herein being considered.

Structural (Topological) Axioms

The *SIGGS Theory* listed the following axioms that include *structural properties*. [The numbers of the axioms refer to their listing in *Steiner and Maccia*.]

53. If complete connectivity increases, then flexibility increases.

54. If strongness decreases, then wholeness increases.

55. If strongness increases, then hierarchical order decreases.

56. If strongness increases, then flexibility increases.

57. If unilateralness, then hierarchical order.

58. If disconnectivity is greater than some value, then independence increases.

59. If disconnectivity is greater than some value, then segregation increases.

60. If vulnerability increases, then complete connectivity decreases.

61. If passive dependence increases, then centrality increases.

62. If active dependence increases, then centrality decreases.

63. If interdependence increases, then complexity growth increases.

64. If hierarchical order increases, then vulnerability increases and flexibility decreases.

65. If compactness increases, then hierarchical order decreases.

66. If centrality increases, then passive dependence increases.

67. If centrality increases, then active dependence decreases.

68. If centrality is less than some value, then independence increases.

69. If centrality is less than some value, then centrality increases.

70. If wholeness increases and hierarchical order is constant, then integration increases.

71. The limit of the ratio of active dependence to passive dependence as unilateralness increases is equal to 1.

90. If toput increases, then centrality decreases.

91. If feedin decreases, then unilateralness decreases.

92. If feedin less than some value decreases, then hierarchical order decreases.

95. If feedthrough increases, then weakness is less than some value.

96. If toput is close to minimum and fromput increases, then disconnectivity increases.

97. If feedin increases and compatibility is close to minimum, then disconnectivity increases.

98. If storeput increases and (filtration decreases or spillage decreases), then integration increases.

100. If complete connectivity increases, then feedin increases.

101. If weakness is greater than some value, then feedthrough is less than some value.

102. If interdependence increases, then feedin increases.

103. If wholeness increases, then regulation is less than some value.

104. If compactness greater than some value increases, then efficiency increases.

105. If centrality increases, then toput decreases.

106. If complete connectivity increases or strongness increases, then toput increases.

107. If complete connectivity increases or strongness increases, then input increases.

108. If complete connectivity increases or strongness increases, then filtration decreases.

109. If complete connectivity increases or strongness increases, then spillage increases.

110. If complete connectivity increases or strongness increases, then 0 is less than change in fromput, and change in fromput is less than change in input.

111. If complete connectivity increases or strongness increases, then change in storeput is greater than change in fromput.

112. If strongness increases and hierarchical order is constant, then regulation decreases.

113. If wholeness increases and hierarchical order is constant, then efficiency decreases.

114. If weakness and hierarchical order, then flexibility decreases.

115. If unilateralness, or weakness increases, or disconnectivity increases, then input decreases and fromput decreases.

137. If feedout is greater than some value and compatibility is less than some value, then segregation is less than some value.

138. If toput increases and compactness greater than some value increases then regulation increases.

139. If toput increases and it is not the case that compactness greater than some value increases, then efficiency decreases.

140. If (fromput is constant or fromput decreases) and complete connectivity increases and strongness increases, then feedthrough decreases.

144. If filtration decreases, then isomorphism increases.

150. If automorphism increases, then input increases and storeput increases and fromput decreases and feedout decreases and filtration decreases and spillage decreases and efficiency decreases.

158. If toput increases and size is constant, then feedback increases.

160. If toput increases and fromput increases and size is constant, then feedout increases.

161. If output is constant and automorphism decreases and homomorphism is greater than some value, then feedout decreases.

164. If independence increases, then stability is less than some value.

165. If flexibility decreases, then state determinacy increases.

166. If m centrality increases, then state steadiness increases.

167. If complexity greater than some value increases, then size increases.

168. If independence increases and wholeness increases, then state steadiness is greater than some value.

169. If wholeness is greater than some value and centrality is greater than some value, then state determinacy is greater than some value.

172. If automorphism increases, then wholeness decreases.

173. If automorphism increases, then centrality decreases.

174. Change in educational system size is greater than change in hierarchical order.

175. If complexity degeneration increases, then size degeneration increases or disconnectivity increases.

176. If state steadiness is less than some value, then segregation is less than some value and integration is less than some value and homeostasis is less than some value.

177. If weakness is maximum and size increases, then passive dependence increases or active dependence increases.

178. If hierarchical order at a given time is greater than some value and size at a given time is greater than some value, then independence at a later time increases.

179. If size increases and complexity growth is constant, then vulnerability increases.

180. If size increases and complexity growth is constant, then flexibility decreases.

181. If size increases and complexity growth is constant, then centrality decreases.

182. If size is constant and complexity degeneration increases, then disconnectivity increases.

183. If size decreases and complexity degeneration increases, then disconnectivity decreases.

184. If complexity increases and size growth is constant, then compactness decreases.

185. If complexity increases and size growth is constant, then centrality increases.

186. If centrality increases and stress is greater than some value, then stability decreases.

187. If stress is equal to 0 and centrality increases, then stability increases.

188. If size increases and complexity growth is constant, then state determinacy increases.

190. If homomorphism at time 2 is greater than homomorphism at time 1, then toput is nearly maximum and size degeneration is nearly maximum and complexity degeneration is nearly maximum.

191. If efficiency is greater than some value and compactness is greater than some value, then state determinacy is greater than some value.

194. If size increases and complexity growth is constant, then toput increases.

195. If size increases and complexity growth is constant, then feedin decreases.

196. If size increases and complexity growth is constant, then feedout increases and change in feedout decreases.

197. If size increases and complexity growth is constant, then feedthrough increases.

198. If size increases and complexity growth is constant, then feedback decreases.

199. If size increases and complexity growth is constant, then regulation increases to some value and then decreases.

200. If size increases and complexity growth is constant, then compatibility decreases.

201. If size increases and complexity growth is constant, then efficiency increases to some value and then decreases.

Formalization of the *ATIS Axioms*

The formalization of the preceding axioms is given below.

53. ${}_{C}\mathscr{C}^{\uparrow} \supset {}_{F}\mathscr{C}^{\uparrow}$

54. ${}_{S}\mathscr{C}^{\downarrow} \supset \mathcal{W}^{\uparrow}$

55. ${}_{S}\mathscr{C}^{\uparrow} \supset {}_{HO}\mathscr{C}^{\downarrow}$

56. ${}_{S}\mathscr{C}^{\uparrow} \supset {}_{F}\mathscr{C}^{\uparrow}$

57. ${}_{U}\mathscr{C} \supset {}_{HO}\mathscr{C}$

58. ${}_{D}\mathscr{C} > \mathrm{n} \supset {}_{I}\mathscr{C}^{\uparrow}$

59. ${}_{D}\mathscr{C} > \mathrm{n} \supset {}_{SG}\mathscr{C}^{\uparrow}$

60. ${}_{V}\mathscr{C}^{\uparrow} \supset {}_{C}\mathscr{C}^{\downarrow}$

61. ${}_{PD}\mathscr{C}^{\uparrow} \supset {}_{C}\mathscr{C}^{\uparrow}$

62. ${}_{AD}\mathscr{C}^{\uparrow} \supset {}_{C}\mathscr{C}^{\downarrow}$

63. ${}_{I}\mathscr{C}^{\uparrow} \supset \mathcal{X}^{+\uparrow}$

64. ${}_{HO}\mathscr{C}^{\uparrow} \supset {}_{V}\mathscr{C}^{\uparrow} \wedge {}_{F}\mathscr{C}^{\downarrow}$

65. ${}_{CP}\mathscr{C}^{\uparrow} \supset {}_{HO}\mathscr{C}^{\downarrow}$

66. ${}_{C}\mathscr{C}^{\uparrow} \supset {}_{PD}\mathscr{C}^{\uparrow}$

67. ${}_{C}\mathscr{C}^{\uparrow} \supset {}_{AD}\mathscr{C}^{\downarrow}$

68. ${}_{C}\mathscr{C} < \mathrm{n} \supset {}_{I}\mathscr{C}^{\uparrow}$

69. ${}_{C}\mathscr{C} < \mathrm{n} \supset {}_{C}\mathscr{C}^{\uparrow}$

70. $\mathcal{W}^{\uparrow} \wedge {}_{HO}\mathscr{C} = \mathrm{c} \supset {}_{IG}\mathscr{C}^{\uparrow}$

71. $\lim_{{}_{U}\mathscr{C}\uparrow}({}_{AD}\mathscr{C} / {}_{PD}\mathscr{C}) = 1$

90. $\mathbf{T}\mathcal{P}^{\uparrow} \supset {}_{C}\mathscr{C}^{\downarrow}$

91. $\boldsymbol{f}_{\mathrm{I}}^{\downarrow} \supset {}_{U}\mathscr{C}^{\downarrow}$

92. $(\boldsymbol{f}_{I} < n)^{\downarrow} \supset {}_{HO}\mathcal{C}^{\downarrow}$

95. $\boldsymbol{f}_{T}^{\uparrow} \supset {}_{W}\mathcal{C} < n$

96. $\mathbf{T}\mathcal{P}^{\approx \min} \wedge \mathbf{F}\mathcal{P}^{\uparrow} \supset {}_{D}\mathcal{C}^{\uparrow}$

97. $\boldsymbol{f}_{I}^{\uparrow} \wedge \boldsymbol{e}^{\approx \min} \supset {}_{D}\mathcal{C}^{\uparrow}$

98. $\mathbf{S}\mathcal{P}^{\uparrow} \wedge ({}_{F}\mathcal{R}^{\downarrow} \vee {}_{S}\mathcal{R}^{\downarrow}) \supset {}_{IG}\mathcal{C}^{\uparrow}$

100. ${}_{C}\mathcal{C}^{\uparrow} \supset \boldsymbol{f}_{I}^{\uparrow}$

101. ${}_{W}\mathcal{C} > n \supset \boldsymbol{f}_{T} < m$

102. ${}_{I}\mathcal{C}^{\uparrow} \supset \boldsymbol{f}_{I}^{\uparrow}$

103. $\mathcal{W}^{\uparrow} \supset {}_{R}\mathcal{R} < n$

104. $({}_{CP}\mathcal{C} > n)^{\uparrow} \supset {}_{EF}\mathcal{S}^{\uparrow}$

105. ${}_{C}\mathcal{C}^{\uparrow} \supset \mathbf{T}\mathcal{P}^{\downarrow}$

106. ${}_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow} \supset \mathbf{T}\mathcal{P}^{\uparrow}$

107. ${}_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow} \supset \mathbf{I}\mathcal{P}^{\uparrow}$

108. ${}_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow} \supset {}_{F}\mathcal{R}^{\downarrow}$

109. ${}_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow} \supset {}_{S}\mathcal{R}^{\uparrow}$

110. ${}_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow} \supset 0 < \Delta\mathbf{F}\mathcal{P} < \Delta\mathbf{I}\mathcal{P}$

111. ${}_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow} \supset \Delta\mathbf{S}\mathcal{P} > \Delta\mathbf{F}\mathcal{P}$

112. ${}_{S}\mathcal{C}^{\uparrow} \wedge {}_{HO}\mathcal{C}_{c} \supset {}_{R}\mathcal{R}^{\downarrow}$

113. $\mathcal{W}^{\uparrow} \wedge {}_{HO}\mathcal{C}_{c} \supset {}_{EF}\mathcal{S}^{\downarrow}$

114. ${}_{W}\mathcal{C} \wedge {}_{HO}\mathcal{C} \supset {}_{F}\mathcal{C}^{\downarrow}$

115. ${}_{U}\mathcal{C}^{\uparrow} \vee {}_{W}\mathcal{C}^{\uparrow} \vee {}_{D}\mathcal{C}^{\uparrow} \supset \mathbf{I}\mathcal{P}^{\downarrow} \wedge \mathbf{F}\mathcal{P}^{\downarrow}$

137. $\boldsymbol{f}_{O} > n \wedge \boldsymbol{e} < m \supset {}_{SG}\mathcal{C} < p$

138. $\mathbf{T}\mathcal{P}^{\uparrow} \wedge ({}_{CP}\mathcal{C} > n)^{\uparrow} \supset {}_{R}\mathcal{R}^{\uparrow}$

139. $\mathbf{T}\mathcal{P}^{\uparrow} \wedge (({}_{CP}\mathcal{C} > n)^{\downarrow} \vee ({}_{CP}\mathcal{C} > n)_c) \supset {}_{EF}\mathcal{S}^{\downarrow}$

140. $(\mathbf{F}\mathcal{P}_c \vee \mathbf{F}\mathcal{P}^{\downarrow}) \wedge {}_{C}\mathcal{C}^{\uparrow} \wedge {}_{S}\mathcal{C}^{\uparrow} \supset \boldsymbol{f}_T^{\downarrow}$

144. ${}_{F}\mathcal{R}^{\downarrow} \supset \mathscr{I}^{\uparrow}$

150. $\mathscr{A}^{\uparrow} \supset \mathbf{I}\mathcal{P}^{\uparrow} \wedge \mathbf{S}\mathcal{P}^{\uparrow} \wedge \mathbf{F}\mathcal{P}^{\downarrow} \wedge \boldsymbol{f}_O^{\downarrow} \wedge {}_{F}\mathcal{R}^{\downarrow} \wedge {}_{S}\mathcal{R}^{\downarrow} \wedge {}_{EF}\mathcal{S}^{\downarrow}$

158. $\mathbf{T}\mathcal{P}^{\uparrow} \wedge \mathcal{Z}_c \supset \boldsymbol{f}_B^{\uparrow}$

160. $\mathbf{T}\mathcal{P}^{\uparrow} \wedge \mathbf{F}\mathcal{P}^{\uparrow} \wedge \mathcal{Z}_c \supset \boldsymbol{f}_O^{\uparrow}$

161. $\mathbf{O}\mathcal{P}_c \wedge \mathscr{A}^{\downarrow} \wedge \mathscr{M} > n \supset \boldsymbol{f}_O^{\downarrow}$

164. ${}_{I}\mathcal{C}^{\uparrow} \supset {}_{SB}\mathfrak{S} < n$

165. ${}_{F}\mathcal{C}^{\downarrow} \supset {}_{D}\mathcal{S}^{\uparrow}$

166. ${}_{C}\mathcal{C}^{\uparrow} \supset {}_{S}\mathcal{S}^{\uparrow}$

167. $(\mathcal{X}^{+} > n)^{\uparrow} \supset \mathcal{Z}^{\uparrow}$

168. ${}_{I}\mathcal{C}^{\uparrow} \wedge \mathcal{W}^{\uparrow} \supset {}_{S}\mathcal{S} > n$

169. $\mathcal{W} > n \wedge {}_{C}\mathcal{C} > m \supset {}_{D}\mathcal{S} > p$

172. $\mathscr{A}^{\uparrow} \supset \mathcal{W}^{\downarrow}$

173. $\mathscr{A}^{\uparrow} \supset {}_{C}\mathcal{C}^{\downarrow}$

174. $\Delta\mathcal{Z} > \Delta_{HO}\mathcal{C}$

175. $\mathcal{X}^{-\uparrow} \supset \mathcal{Z}^{-\uparrow} \vee {}_{D}\mathcal{C}^{\uparrow}$

176. ${}_{S}\mathcal{S} < n \supset {}_{SG}\mathcal{C} < m \wedge {}_{IG}\mathcal{C} < p \wedge {}_{H}\mathfrak{S} < r$

177. ${}_{W}\mathcal{C}^{max} \wedge \mathcal{Z}^{\uparrow} \supset {}_{PD}\mathcal{C}^{\uparrow} \vee {}_{AD}\mathcal{C}^{\uparrow}$

178. ${}_{HO}\mathcal{C}(t_1) > n \wedge \mathcal{Z}(t_1) > m \supset {}_{I}\mathcal{C}(t_2)^{\uparrow}$

179. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}_c \supset {}_{V}\mathcal{C}^{\uparrow}$

180. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}_c \supset {}_{F}\mathcal{C}^{\downarrow}$

181. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}_c \supset {}_{C}\mathcal{C}^{\downarrow}$

182. $\mathcal{Z}_c \wedge \mathcal{X}^{-\uparrow} \supset {}_{D}\mathcal{C}^{\uparrow}$

183. $\mathcal{Z}^{\downarrow} \wedge \mathcal{X}^{-\uparrow} \supset {}_{D}\mathcal{C}^{\downarrow}$

184. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c} \supset {}_{CP}\mathcal{C}^{\downarrow}$

185. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c} \supset {}_{C}\mathcal{C}^{\uparrow}$

186. $\mathcal{X}^{+\uparrow} \wedge {}_{ST}\mathbb{S}' > n \supset {}_{SB}\mathbb{S}^{\downarrow}$

187. ${}_{ST}\mathbb{S}' = 0 \wedge {}_{C}\mathcal{C}^{\uparrow} \supset {}_{SB}\mathbb{S}^{\uparrow}$

188. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset {}_{D}\mathcal{S}^{\uparrow}$

190. $\mathscr{M}(t_2) > \mathscr{M}(t_1) \supset \mathbf{T}\mathcal{P}^{\approx \max} \wedge \mathcal{Z}^{-\approx \max} \wedge \mathcal{X}^{-\approx \max}$

191. ${}_{EF}\mathcal{S} > n \wedge {}_{CP}\mathcal{C} > m \supset {}_{D}\mathcal{S} > p$

194. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \mathbf{T}\mathcal{P}^{\uparrow}$

195. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \boldsymbol{f}_{I}{}^{\downarrow}$

196. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \boldsymbol{f}_{O}{}^{\uparrow} \wedge \Delta\boldsymbol{f}_{O}{}^{\downarrow}$

197. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \boldsymbol{f}_{T}{}^{\uparrow}$

198. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \boldsymbol{f}_{B}{}^{\downarrow}$

199. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \varphi({}_{R}\mathcal{R}^{\uparrow})^{\max} = n_{t(1)} \wedge \varphi({}_{R}\mathcal{R}^{\downarrow})^{\min} = m_{t(2)} \wedge m < n$; where φ is a measure of ${}_{R}\mathcal{R}$.

200. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \boldsymbol{e}^{\downarrow}$

201. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset \phi({}_{EF}\mathcal{S}^{\uparrow})^{\max} = n_{t(1)} \wedge \phi({}_{EF}\mathcal{S}^{\downarrow})^{\min} = m_{t(2)} \wedge m < n$; where ϕ is a measure of ${}_{EF}\mathcal{S}$.

The First-Order Theorems

The following 27 theorems are derived from the one-step transitive property of implication, shown by *Logical Schema 0* below. They are numbered by adjoining the logical schema number with the numbers of the axioms from which they are derived and in the order of the transitivity.

Logical Schema 0. $\mathbf{P \supset Q, Q \supset R \vdash P \supset R}$

[**NOTE**: The symbol '⊢' is used, as it is conventionally, for "yields". Further, it is to be understood that the statements left of '⊢' are "assumptions"; that is, they are considered to be valid. Since we are initially concerned with the axioms of the theory, they are all valid.]

T.0.150-144. $\mathscr{A}^{\uparrow} \supset \mathscr{I}^{\uparrow}$

If system automorphism increases, then isomorphism increases.

That is, the transitivity schema is applied to Axiom 150, $\mathscr{A}^{\uparrow} \supset \mathbf{I}_{\mathcal{P}}{}^{\uparrow} \wedge \mathbf{S}_{\mathcal{P}}{}^{\uparrow} \wedge \mathbf{F}_{\mathcal{P}}{}^{\downarrow} \wedge \boldsymbol{f}_{\mathrm{O}}{}^{\downarrow} \wedge {}_{\mathrm{F}}\mathcal{R}^{\downarrow} \wedge {}_{\mathrm{S}}\mathcal{R}^{\downarrow} \wedge {}_{\mathrm{EF}}\mathcal{S}^{\downarrow}$, and Axiom 144, ${}_{\mathrm{F}}\mathcal{R}^{\downarrow} \supset \mathscr{I}^{\uparrow}$ to obtain Theorem 0.150-144: $\mathscr{A}^{\uparrow} \supset \mathscr{I}^{\uparrow}$. The following theorems are obtained in a similar manner.

T.0.106-90. ${}_{\mathrm{C}}\mathscr{C}^{\uparrow} \vee {}_{\mathrm{S}}\mathscr{C}^{\uparrow} \supset {}_{\mathrm{C}}\mathscr{C}^{\downarrow}$

If system complete connectivity increases or strongness increases, then centrality decreases.

T.0.108-144. ${}_{\mathrm{C}}\mathscr{C}^{\uparrow} \vee {}_{\mathrm{S}}\mathscr{C}^{\uparrow} \supset \mathscr{I}^{\uparrow}$

If system complete connectivity increases or strongness increases, then isomorphism increases.

T.0.66-61. ${}_{\mathrm{C}}\mathscr{C}^{\uparrow} \equiv {}_{\mathrm{PD}}\mathscr{C}^{\uparrow}$

System centrality increases, if and only if passive dependence increases.

T.0.68-63. ${}_{\mathrm{C}}\mathscr{C} < \mathrm{n} \supset \mathscr{X}^{+\uparrow}$

If system centrality is less than some value, then complexity growth increases.

T.0.68-102. ${}_{\mathrm{C}}\mathscr{C} < \mathrm{n} \supset \boldsymbol{f}_{\mathrm{I}}{}^{\uparrow}$

If system centrality is less than some value, then feedin increases.

T.0.68-164. ${}_{\mathrm{C}}\mathscr{C} < \mathrm{n} \supset {}_{\mathrm{SB}}\mathfrak{S} < \mathrm{n}$

If system centrality is less than some value, then stability is less than some value.

T.0.69-66. ${}_{\mathrm{C}}\mathscr{C} < \mathrm{n} \supset {}_{\mathrm{PD}}\mathscr{C}^{\uparrow}$

If system centrality is less than some value, then passive dependence increases.

T.0.66-61/69-66. $_{C}\mathcal{C} < n \supset {}_{C}\mathcal{C}^{\uparrow}$

If system centrality is less than some value, then centrality increases.

T.0.69-67. $_{C}\mathcal{C} < n \supset {}_{AD}\mathcal{C}^{\downarrow}$

If system centrality is less than some value, then active dependence decreases.

T.0.69-105. $_{C}\mathcal{C} < n \supset \mathbf{T}\mathcal{P}^{\downarrow}$

If system centrality is less than some value, then toput decreases.

T.0.69-166. $_{C}\mathcal{C} < n \supset {}_{S}\mathcal{S}^{\uparrow}$

If system centrality is less than some value, then state steadiness increases.

T.0.185-66. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c} \supset {}_{PD}\mathcal{C}^{\uparrow}$

If system complexity increases and size growth is constant, then passive dependence increases.

T.0.66-61/185-66. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c} \supset {}_{C}\mathcal{C}^{\uparrow}$

If system complexity increases and size growth is constant, then centrality increases.

T.0.185-67. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c} \supset {}_{AD}\mathcal{C}^{\downarrow}$

If system complexity increases and size growth is constant, then active dependence decreases.

T.0.185-105. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c} \supset \mathbf{T}\mathcal{P}^{\downarrow}$

If system complexity increases and size growth is constant, then toput decreases.

T.0.185-166. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c} \supset {}_{S}\mathcal{S}^{\uparrow}$

If system complexity increases and size growth is constant, then state steadiness increases.

T.0.58-63. $_{D}\mathcal{C} > n \supset \mathcal{X}^{+\uparrow}$

If system disconnectivity is greater than some value, then complexity growth increases.

T.0.58-102. $_{D}\mathcal{C} > n \supset \boldsymbol{f}_{I}^{\uparrow}$

If system disconnectivity is greater than some value, then feedin increases.

T.0.58-164. $_{D}\mathcal{C} > n \supset {}_{SB}\mathfrak{S} < m$

If system disconnectivity is greater than some value, then stability is less than some value.

T.0.61-67. $_{PD}\mathcal{C}^{\uparrow} \supset {}_{AD}\mathcal{C}^{\downarrow}$

If system passive dependence increases, then active dependence decreases.

T.0.61-105. $_{PD}\mathcal{C}^{\uparrow} \supset \mathbf{T}\mathcal{P}^{\downarrow}$

If system passive dependence increases, then toput decreases.

T.0.61-166. $_{PD}\mathcal{C}^{\uparrow} \supset {}_{S}\mathcal{S}^{\uparrow}$

If system passive dependence increases, then state steadiness increases.

T.0.114-165. $_{W}\mathcal{C} \wedge {}_{HO}\mathcal{C} \supset {}_{D}\mathcal{S}^{\uparrow}$

If system weakness and hierarchical order, then state determinacy increases.

T.0.54-103. $_{S}\mathcal{C}\downarrow \supset {}_{R}\mathcal{R} < n$

If system strongness decreases, then regulation is less than some value.

T.0.179-60. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset {}_{C}\mathcal{C}^{\downarrow}$

If system size increases and complexity growth is constant, then complete connectivity decreases.

T.0.194-90. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset {}_{C}\mathcal{C}^{\downarrow}$

If system size increases and complexity growth is constant, then centrality decreases.

T.0.195-91. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset {}_{U}\mathcal{C}^{\downarrow}$

If system size increases and complexity growth is constant, then unilateralness decreases.

T.0.197-95. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c} \supset {}_{W}\mathcal{C} < n$

If system size increases and complexity growth is constant, then weakness is less than some value.

Additional Theorems

The theorems in this section are derived from additional theorems of the statement calculus. The theorems of the statement calculus will not be proved, but simply cited. Then, the theorems of the present theory derived from these statement calculus theorems will be given. These theorems will help to develop the theory and show relations not otherwise obvious. The theorems from the statement calculus will be presented as "logical schemas" in which properties from the current theory will be substituted for the variables in the schema.

The following 26 theorems are derived from the following logical schema.

Logical Schema 1. $\mathbf{P \supset Q, R \supset Q \vdash P \vee R \supset Q.}$

T.1.53-56. $_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow} \supset {}_{F}\mathcal{C}^{\uparrow}$

If system complete connectivity increases or strongness increases, then flexibility increases.

T.1.55-65. $_{S}\mathcal{C}^{\uparrow} \vee {}_{CP}\mathcal{C}^{\uparrow} \supset {}_{HO}\mathcal{C}^{\downarrow}$

If system strongness increases or compactness increases, then hierarchical order decreases.

T.1.55-92. $_{S}\mathcal{C}^{\uparrow} \vee (f_{I} < n)^{\downarrow} \supset {}_{HO}\mathcal{C}^{\downarrow}$

If system strongness or feedin less than some value decreases, then hierarchical order decreases.

T.1.58-68. $_{D}\mathscr{C} > n \vee {}_{C}\mathscr{C} < m \supset {}_{I}\mathscr{C}^{\uparrow}$

If system disconnectivity is greater than some value or centrality is less than some value, then independence increases.

T.1.61-69. $_{PD}\mathscr{C}^{\uparrow} \vee {}_{C}\mathscr{C} < n \supset {}_{C}\mathscr{C}^{\uparrow}$

If system passive dependence increases or centrality is less than some value, then centrality increases.

T.1.61-185. $_{PD}\mathscr{C}^{\uparrow} \vee [\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c}] \supset {}_{C}\mathscr{C}^{\uparrow}$

If system passive dependence increases, or complexity increases and size growth is constant, then centrality increases.

T.1.62-173. $_{AD}\mathscr{C}^{\uparrow} \vee \mathscr{A}^{\uparrow} \supset {}_{C}\mathscr{C}^{\downarrow}$

If system active dependence increases or automorphism increases, then centrality decreases.

T.1.62-90. $_{AD}\mathscr{C}^{\uparrow} \vee \mathbf{T}\mathcal{P}^{\uparrow} \supset {}_{C}\mathscr{C}^{\downarrow}$

If system active dependence increases or toput increases, then centrality decreases.

T.1.62-173. $_{AD}\mathscr{C}^{\uparrow} \vee \mathscr{A}^{\uparrow} \supset {}_{C}\mathscr{C}^{\downarrow}$

If system active dependence increases or automorphism increases, then centrality decreases.

T.1.62-181. $_{AD}\mathscr{C}^{\uparrow} \vee [\,\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c}] \supset {}_{C}\mathscr{C}^{\downarrow}$

If system active dependence increases, or size increases and complexity growth is constant, then centrality decreases.

T.1.64-114. $_{HO}\mathscr{C}^{\uparrow} \vee [{}_{W}\mathscr{C} \wedge {}_{HO}\mathscr{C}] \supset {}_{F}\mathscr{C}^{\downarrow}$

If system hierarchical order increases, or weakness and hierarchical order, then flexibility decreases.

T.1.64-180. $_{HO}\mathscr{C}^{\uparrow} \vee [\,\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c}] \supset {}_{F}\mathscr{C}^{\downarrow}$

If system hierarchical order increases, or size increases and complexity growth is constant, then flexibility decreases.

T.1.65-92. $_{CP}\mathscr{C}^{\uparrow} \vee (f_{I} < n)^{\downarrow} \supset {}_{HO}\mathscr{C}^{\downarrow}$

If system compactness increases or feedin less than some value decreases, then hierarchical order decreases.

T.1.69-185. $_{C}\mathscr{C} < n \vee [\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_{c}] \supset {}_{C}\mathscr{C}^{\uparrow}$

If system centrality is less than some value, or complexity increases and size growth is constant, then centrality increases.

T.1.70-98. $[\mathcal{W}^{\uparrow} \wedge {}_{HO}\mathscr{C}_{c}] \vee [\mathbf{S}\mathcal{P}^{\uparrow} \wedge ({}_{F}\mathcal{R}^{\downarrow} \vee {}_{S}\mathcal{R}^{\downarrow})] \supset {}_{IG}\mathscr{C}^{\uparrow}$

If system wholeness increases and hierarchical order is constant, or storeput increases and (filtration decreases or spillage decreases), then integration increases.

T.1.90-173. $\mathbf{T}\mathcal{P}^{\uparrow} \vee \mathscr{A}^{\uparrow} \supset {}_{C}\mathscr{C}^{\downarrow}$

If system toput increases or automorphism increases, then centrality decreases.

T.1.90-181. $\mathbf{T}\mathcal{P}^{\uparrow} \vee [\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c}] \supset {}_{C}\mathcal{C}^{\downarrow}$

If system toput increases, or size increases and complexity growth is constant, then centrality decreases.

T.1.96-97. $[\mathbf{T}\mathcal{P}^{\approx \min} \wedge \mathbf{F}\mathcal{P}^{\uparrow}] \vee [\boldsymbol{f}_{I}^{\uparrow} \wedge \boldsymbol{c}^{\approx \min}] \supset {}_{D}\mathcal{C}^{\uparrow}$

If system toput is close to minimum and fromput increases, or feedin increases and compatibility is close to minimum, then disconnectivity increases.

T.1.96-182. $[\mathbf{T}\mathcal{P}^{\approx \min} \wedge \mathbf{F}\mathcal{P}^{\uparrow}] \vee [\mathcal{Z} = c \wedge \mathcal{X}^{-\uparrow}] \supset {}_{D}\mathcal{C}^{\uparrow}$

If system toput is nearly minimum and fromput increases, or size is constant and complexity degeneration increases, then disconnectivity increases.

T.1.97-182. $[\boldsymbol{f}_{I}^{\uparrow} \wedge \boldsymbol{c}^{\approx \min}] \vee [\mathcal{Z}_{c} \wedge \mathcal{X}^{-\uparrow}] \supset {}_{D}\mathcal{C}^{\uparrow}$

If system feedin increases and compatibility is close to minimum, or size is constant and complexity degeneration increases, then disconnectivity increases.

T.1.100-102. ${}_{C}\mathcal{C}^{\uparrow} \vee {}_{I}\mathcal{C}^{\uparrow} \supset \boldsymbol{f}_{I}^{\uparrow}$

If system complete connectivity increases or interdependence increases, then feedin increases.

T.1.106-194. $[{}_{C}\mathcal{C}^{\uparrow} \vee {}_{S}\mathcal{C}^{\uparrow}] \vee [\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c}] \supset \mathbf{T}\mathcal{P}^{\uparrow}$

If system complete connectivity increases or strongness increases, or size increases and complexity growth is constant, then toput increases.

T.1.113-139. $[\mathcal{W}^{\uparrow} \wedge {}_{HO}\mathcal{C}_{c}] \vee [\mathbf{T}\mathcal{P}^{\uparrow} \wedge (({}_{CP}\mathcal{C} > n)^{\downarrow} \vee ({}_{CP}\mathcal{C} > n)_{c})] \supset {}_{EF}\mathcal{S}^{\downarrow}$

If system wholeness increases and hierarchical order is constant, or toput increases and compactness greater than some value increases or remains constant, then efficiency decreases.

T.1.114-180. $[{}_{W}\mathcal{C} \wedge {}_{HO}\mathcal{C}] \vee [\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c}] \supset {}_{F}\mathcal{C}^{\downarrow}$

If system weakness and hierarchical order, or size increases and complexity growth is constant, then flexibility decreases.

T.1.165-188. ${}_{F}\mathcal{C}^{\downarrow} \vee [\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c}] \supset {}_{D}\mathcal{S}^{\uparrow}$

If system flexibility decreases, or size increases and complexity growth is constant, then state determinacy increases.

T.1.173-181. $\mathscr{A}^{\uparrow} \vee [\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_{c}] \supset {}_{C}\mathcal{C}^{\downarrow}$

If system automorphism increases, or size increases and complexity growth is constant, then centrality decreases.

Logical Schema 2. **P ⊃ Q, R ⊃ S ⊢ PR ⊃ QS**

These may prove to be useful theorems when developing various paths of connectedness within the theory. Only one example will be given below, since, obviously, this theorem will generate thousands of theorems in the present theory. In fact, with only the selected axioms currently being considered, far short of those given by Steiner and Maccia, the number of theorems generated by this *Logical Schema 2* is: $\sum_{n=1...81}(n-1) = 3{,}240$.

T.2.53-66. $_{C}\mathcal{O}^{\uparrow} \supset {}_{F}\mathcal{O}^{\uparrow},\ {}_{C}\mathcal{O}^{\uparrow} \supset {}_{PD}\mathcal{O}^{\uparrow} \vdash {}_{C}\mathcal{O}^{\uparrow} \wedge {}_{C}\mathcal{O}^{\uparrow} \supset {}_{F}\mathcal{O}^{\uparrow} \wedge {}_{PD}\mathcal{O}^{\uparrow}$

The Power of Logical Schemas in Theory Development

There are several logical schemas that will prove to be extremely beneficial in developing the connectedness of various relations within the theory. The power of these schemas will become even more apparent when applied in conjunction with various topological analyses discussed later. For now, *Logical Schema 3*, shown below, will give an indication of just how far-reaching these analyses will be.

Logical Schema 3. $\mathbf{P} \supset \mathbf{R} \vdash \mathbf{P} \supset (\mathbf{Q} \supset \mathbf{R})$

The following application exemplifies the value of this schema.

T.3.67. $_{\mathrm{C}}\mathscr{O}^{\uparrow} \supset {}_{\mathrm{AD}}\mathscr{O}^{\downarrow} \vdash {}_{\mathrm{C}}\mathscr{O}^{\uparrow} \supset (\mathbf{X} \supset {}_{\mathrm{AD}}\mathscr{O}^{\downarrow})$

Assumption: If system centrality increases, then active dependence decreases (Axiom 67).

Given: X is any other (verified) system property.

The Statement Formula:

If system centrality increases, then if X then active dependence decreases.

Assume that **X** is a property describing the system. Then you can conclude that if centrality increases, then active dependence will decrease as a result of **X**.

This should prove to be a very strong theorem in helping to analyze the connectedness, and control of the connectedness, of the system. Let's see how this might work when applied to a particular system.

Given: Education System $\mathbb{S}_{\mathrm{E}}$, and the empirical property $\boldsymbol{f}_{\mathrm{I}}^{\downarrow} = \mathbf{X}$. [That is, it has been empirically verified that in System $\mathbb{S}_{\mathrm{E}}$, $\boldsymbol{f}_{\mathrm{I}}^{\downarrow}$ when $_{\mathrm{C}}\mathscr{O}^{\uparrow}$; that is, feedin decreases when centrality increases.]

Theorem: $_{\mathrm{C}}\mathscr{O}^{\uparrow} \supset {}_{\mathrm{AD}}\mathscr{O}^{\downarrow} \vdash {}_{\mathrm{C}}\mathscr{O}^{\uparrow} \supset (\boldsymbol{f}_{\mathrm{I}}^{\downarrow} \supset {}_{\mathrm{AD}}\mathscr{O}^{\downarrow})$

Application:

(1)	$_{\mathrm{C}}\mathscr{O}^{\uparrow} \supset {}_{\mathrm{AD}}\mathscr{O}^{\downarrow}$	Assumption
(2)	$\boldsymbol{f}_{\mathrm{I}}^{\downarrow}$	Assumption
(3)	$\vdash {}_{\mathrm{C}}\mathscr{O}^{\uparrow} \supset (\boldsymbol{f}_{\mathrm{I}}^{\downarrow} \supset {}_{\mathrm{AD}}\mathscr{O}^{\downarrow})$	Specification of Schema

Analysis:

$_{AD}\mathscr{O}^{\downarrow}$ is true. It is desired to have $_{AD}\mathscr{O}^{\uparrow}$. However, that will make the conclusion of the implication false. That cannot hold in the system if $\boldsymbol{f}_{I}^{\downarrow}$ is true, since the implication, $\boldsymbol{f}_{I}^{\downarrow} \supset {}_{AD}\mathscr{O}^{\downarrow}$, would be false. Therefore, $_{AD}\mathscr{O}^{\uparrow}$ implies $\boldsymbol{f}_{I}^{\uparrow}$ in order for the implication to be true. Therefore, to obtain $_{AD}\mathscr{O}^{\uparrow}$, change the feedin of the system so that it increases. (This analysis is actually an application of *Logical Schemas 5* and 6.) Thus, we have an example that demonstrates that *ATIS* is predictive, and provides non-obvious solutions for specific problems.

Additional theorems will be given below, however, no detailed analysis will be made of them at this time. The intent at this point is to simply provide a list of the theorems that may be applicable to any particular education system.

Logical Schema 4. $\mathbf{P \supset Q, P \supset R \vdash P \supset QR}$

This theorem can be used to simplify representations within certain systems to enhance understanding and achieve greater facility in establishing the connectedness of the system. This logical schema demonstrates that there are several applications of theorems that, while they are not substantively fruitful, they will provide the means to develop patterns within a system that might otherwise not be recognized.

T.4.53-100. $_{C}\mathscr{O}^{\uparrow} \supset {}_{F}\mathscr{O}^{\uparrow}, {}_{C}\mathscr{O}^{\uparrow} \supset \boldsymbol{f}_{I}^{\uparrow} \vdash {}_{C}\mathscr{O}^{\uparrow} \supset {}_{F}\mathscr{O}^{\uparrow} \wedge \boldsymbol{f}_{I}^{\uparrow}$

That is, given Axioms 53 and 100, the following statement formula can be obtained:

$$\vdash {}_{C}\mathscr{O}^{\uparrow} \supset {}_{F}\mathscr{O}^{\uparrow} \wedge \boldsymbol{f}_{I}^{\uparrow}$$

It is noted that the assumptions do not have to be axioms. They are "assumptions." However, they can be any of the axioms or any theorem derived from the axioms as well as assumptions relevant to a particular system.

Logical Schema 5. $\mathbf{Q \supset P \vdash {\sim}P \supset {\sim}Q}$ **<u>or</u>** $\mathbf{Q \supset P \mathrel{.{\equiv}.} {\sim}P \supset {\sim}Q}$

As shown in the *analysis* given in *Logical Schema 3*, this *Logical Schema 5* in conjunction with *Logical Schema 6* may prove to be very useful in **controlling** a given system.

T.5.55. $_{S}\mathscr{O}^{\uparrow} \supset {}_{HO}\mathscr{O}^{\downarrow} \vdash {}_{HO}\mathscr{O}_{c}^{\uparrow} \supset {}_{S}\mathscr{O}_{c}^{\downarrow}$

The notation, $\mathbf{X}_{c}^{\uparrow}$, means that property **X** is constant or increases, which, in this case, is the negation of $\mathbf{X}^{\downarrow}$.

Logical Schema 6. If $P \supset Q$, then $P \vdash Q$

This schema allows you to take an implication and treat the hypothesis as an assumption of a statement formula from which the conclusion is actually derived.

T.6.91. **If** $\vdash \boldsymbol{f}_{I}^{\downarrow} \supset {}_{U}\mathscr{O}^{\downarrow}$**, then** $\boldsymbol{f}_{I}^{\downarrow} \vdash {}_{U}\mathscr{O}^{\downarrow}$

Logical Schema 7. $\vdash \sim(\sim PP)$

This logical schema, while not necessarily substantively fruitful in itself, may provide the means for developing patterns within a system that might otherwise not be recognized, or it may be required in the proof of certain theorems for showing inconsistencies.

T.7.174. $\vdash \sim[\sim(\Delta\mathcal{Z} > \Delta_{HO}\mathscr{O})(\Delta\mathcal{Z} > \Delta_{HO}\mathscr{O})]$

List of Logical Schemas

The following list of the logical schemas is provided to facilitate theorem proofs. The logical schemas given previously are also listed so as to facilitate their use. Further, they are listed according to the form of the schema, rather than the order in which they were discussed. No examples will be given for the new schemas since their substitutions should be obvious.

The "System Construction Theorems" (SCTs), derived directly from the axioms of the Statement Calculus, provide a means of developing the connectedness of a system. These should prove important in developing the system topology.

Logical Schema 6. **If** $P \supset Q$, **then** $P \vdash Q$; and **If** $P \vdash Q$, **then** $P \supset Q$ **..≡..**

$$P \vdash Q \ .\equiv. \ \vdash P \supset Q$$

"$P \vdash Q \ .\supset. \ P \supset Q$" is the ***Deduction Theorem***.

Logical Schema 7. $\vdash \sim(\sim PP)$

Logical Schema 8. $\sim\sim P \ .\equiv. \ P$

Logical Schema 9. $\vdash \sim P \vee P$

Logical Schema 19. $P, P \supset Q \vdash Q$ (Modus Ponens)

Logical Schema 10. $P \vdash Q \supset PQ$ (System Construction Theorem)

Logical Schema 11. $\sim(QR) \vdash R \supset \sim Q$

Logical Schema 3. $P \supset R \vdash P \supset (Q \supset R)$ (System Construction Theorem)

Logical Schema 5. $Q \supset P \ .\equiv. \ \sim P \supset \sim Q$

Logical Schema 12. $P \supset Q \vdash PR \supset QR$ (System Construction Theorem)

Logical Schema 13. $R \supset S \vdash PR \supset PS$ (System Construction Theorem)

Logical Schema 14. $PQ \supset P \vdash P \supset (Q \supset R)$ (System Construction Theorem)

Logical Schema 15. $PQ \supset R \ .\equiv. \ P \supset (Q \supset R)$

Logical Schema 16.	**P ⊃ ~Q ⊢ P ⊃ (Q ⊃ R)**	(System Construction Theorem)
Logical Schema 17.	**P ⊃ ~R ⊢ P ⊃ ~(QR)**	(System Construction Theorem)
Logical Schema 18.	**P ⊃ Q, P ⊃ ~R ⊢ P ⊃ ~(Q ⊃ R)**	
Logical Schema 0.	**P ⊃ Q, Q ⊃ R ⊢ P ⊃ R**	(Transitive Property of ⇒)
Logical Schema 1.	**P ⊃ Q, R ⊃ Q ⊢ P ∨ R ⊃ Q**	
Logical Schema 2.	**P ⊃ Q, R ⊃ S ⊢ PR ⊃ QS**	
Logical Schema 4.	**P ⊃ Q, P ⊃ R ⊢ P ⊃ QR**	

Definitions of Logical Operations

In addition to the logical schemas presented previously, some theorems may require an application of the definition of the logical operations. These will be discussed below.

The statement calculus starts with two undefined operations: $\sim$ and $\wedge$; read as "not" and "and," respectively.

Then, the following operations are defined in terms of the above two undefined operations:

$\vee$, $\supset$, and $\equiv$ read as "or," "implies" [or, "If ... then ..."]; and "if and only if," respectively.

Definition. $P \vee Q \;=_{Df}\; \sim(\sim P \wedge \sim Q)$

Definition. $P \supset Q \;=_{Df}\; \sim(P \wedge \sim Q)$

Definition. $P \equiv Q \;=_{Df}\; (P \supset Q) \wedge (Q \supset P)$ **.≡.** $\sim(P \wedge \sim Q) \wedge \sim(Q \wedge \sim P)$

The following theorem demonstrates an application of the use of definitions in the proof of a theorem.

Theorem. ${}_{HO}\mathscr{C}_c^{\uparrow} \vee {}_{F}\mathscr{C}_c^{\downarrow} \supset {}_{S}\mathscr{C}_c^{\downarrow}$

Proof:

1.	${}_{S}\mathscr{C}^{\uparrow} \supset {}_{HO}\mathscr{C}^{\downarrow}$	Axiom 55
2.	${}_{S}\mathscr{C}^{\uparrow} \supset {}_{F}\mathscr{C}^{\uparrow}$	Axiom 56
3.	$\therefore\; {}_{S}\mathscr{C}^{\uparrow} \supset {}_{HO}\mathscr{C}^{\downarrow} \wedge {}_{F}\mathscr{C}^{\uparrow}$	Logical Schema 4
4.	$\sim({}_{HO}\mathscr{C}^{\downarrow} \wedge {}_{F}\mathscr{C}^{\uparrow}) \supset \sim {}_{S}\mathscr{C}^{\uparrow}$	Logical Schema 5
5.	${}_{HO}\mathscr{C}_c^{\uparrow} \vee {}_{F}\mathscr{C}_c^{\downarrow} \supset \sim {}_{S}\mathscr{C}^{\uparrow}$	Definition of "$\vee$"
6.	$\therefore\; {}_{HO}\mathscr{C}_c^{\uparrow} \vee {}_{F}\mathscr{C}_c^{\downarrow} \supset {}_{S}\mathscr{C}_c^{\downarrow}$	Logical Equivalence of "$\sim$"

Axioms 181 and 184 are Theorems

With the number of axioms presented for the theory, it is possible that some of the axioms are in fact theorems. That is, they are derivable from the other axioms. Such is the case with Axioms 181 and 184.

In the case of Axiom 184, however, we also have that the conclusion of the axiom is shown to be false with respect to the other axioms. Axiom 184 is stated as follows:

$$184. \quad \mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_c \supset {}_{CP}\mathcal{O}^{\downarrow}$$

However, the theorem to be proved is the following: $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_c \supset {}_{CP}\mathcal{O}^{\uparrow}$.

Theorem. $\mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_c \supset {}_{CP}\mathcal{O}^{\uparrow}$

Proof:

(1) $\mathcal{X}^{+\uparrow}$	Assumption
(2) $\mathcal{Z}_c$	Assumption
(3) $\mathcal{Z} =_{df} C(\mathcal{S})$	Definition
(4) $C(\mathcal{S})_c$	Substitution of (3) into (2)
(5) $\mathcal{X}^{+} =_{df} C(\mathbf{P}^{\mathbf{uc}})$	Definition
(6) $C(\mathbf{P}^{\mathbf{uc}})^{\uparrow}$	Substitution of (5) into (1)
(7) ${}_{CP}\mathcal{O} =_{df} C(\mathbf{P}^{\mathbf{pc}})$	Definition
(8) $C(\mathbf{P}^{\mathbf{uc}})^{\uparrow} \supset C(\mathbf{P}^{\mathbf{pc}})^{\uparrow}$	Definition of $C(\mathbf{P}^{\mathbf{pc}})$
(9) $\therefore C(\mathbf{P}^{\mathbf{uc}})^{\uparrow} \wedge C(\mathcal{S})_c$	Assumptions
(10) $C(\mathbf{P}^{\mathbf{pc}})^{\uparrow}$	Modus Ponens on (9) and (8)
(11) $\therefore C(\mathbf{P}^{\mathbf{uc}})^{\uparrow} \wedge C(\mathcal{S})_c \supset C(\mathbf{P}^{\mathbf{pc}})^{\uparrow}$	Statement Calculus
(12) $\therefore \mathcal{X}^{+\uparrow} \wedge \mathcal{Z}_c \supset {}_{CP}\mathcal{O}^{\uparrow}$	Substitution

Axiom 181 states: $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_c \supset {}_{C}\mathcal{O}^{\downarrow}$. This statement will now be proved as a theorem.

Theorem. $\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_c \supset {}_{C}\mathcal{O}^{\downarrow}$

Proof:

1.	$\mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_c \supset \mathbf{T}_{\mathcal{P}}{}^{\uparrow}$	Axiom 194
2.	$\mathbf{T}_{\mathcal{P}}{}^{\uparrow} \supset {}_{C}\mathcal{O}^{\downarrow}$	Axiom 90
3.	$\therefore \mathcal{Z}^{\uparrow} \wedge \mathcal{X}^{+}{}_c \supset {}_{C}\mathcal{O}^{\downarrow}$	Logical Schema 0