Models and the Meaning of 'Retroduction'

by Elizabeth Steiner Maccia

Foundations Division

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Acknowledgment

This paper is the second in a series dealing with the methodology for constructing educational theory models and the rationale thereof, i.e. with the metatheoretical dimension of Project 1632. This research is supported under the Cooperative Research Program of the Office of Education, U. S. Department of Health, Education, and Welfare.

"I'm sure I didn't mean--" Alice was beginning, but the Red Queen interrupted her impatiently.

"That's just what I complain of! You <u>should</u> have meant! What do you suppose is the use of a child without any meaning?"

A help to the mind in developing science without defect which has no, or little, stated meaning surely would lead to supposition regarding its use. I have laid claim to retroduction as a help in devising theory.[1] Retroduction, therefore, must be explicated. The indefinite must be made definite.

Peirce's discussion of retroduction included reference to a point of view that would resolve the wonder arising from pondering phenomena or objects. The point of view allows setting forth a conjecture characterizing the objects.[2] But what is meant by a 'point of view' and how does it relate to the conjecture? A nineteenth century physicist, Heinrich Hertz, indicated the context wherein an answer is to be sought by drawing attention to models.

> We form for ourselves images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured. In order that this requirement may be satisfied, there must be a certain conformity between nature and our thought. Experience teaches us that the requirement can be satisfied, and hence that such a conformity does in fact exist. When from our accumulated previous experience we have once succeeded in deducing images of the desired nature, we can then in a short time develop by means of them, as by means of models, the consequences which in the external world only arise in a comparatively long time, or as the result of our own interposition.[3]

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[1]See "Ways of Inquiring," the first paper in this series, Publication 62-107, The Bureau of Educational Research and Service, The Ohio State University, 1962.

[2]*Values in a Universe of Chance*, ed. by Phillip P. Wiener, New York: Doubleday Anchor Books, 1958, p. 367.

[3]*The Principles of Mechanics*, trans. by D. E. Jones and J. T. Walley, London: Macmillan and Co., 1899, p. 1.

But the context marked off by Hertz needs sorting about, if through the concept of model we are to say anything significant of retroduction. 'Model' has come to have many meanings some of which have nothing, or little, to do with the developing of science without defect. For example, a book entitled Mathematical Models contains 278 pages setting forth directions for constructing physical objects which are taken to be models demonstrative of mathematics. To illustrate, one is told how to construct the Galton Quincunx which demonstrates the binominal distribution.[4] Models demonstrative of science also can be constructed. The familiar friend of school boys, the Cartesian diver, is numbered among these. Indeed the category of demonstrative models includes many of the audio-visual devices of the educator. Of course, one might say that such models have a little to do with the development of science. They might well provide scientific understanding, and so the occasional professor who might develop science.[5] The first sorting, then, is in terms of the domain of the logic of science. Demonstrative models are of little interest here.

Within the domain of the logic of science, there are also

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[4]H. Martyn Cundy and A. P. Rollett, Mathematical Models, Oxford: Clarendon Press, Second edition, 1961, pp. 217-219.

[5]The following interesting footnote in the book might substantiate in part such a claim: "There is a beautiful collection of polyhedral models in wire and cardboard at Winchester College. These were made by three boys, F. J. Dyson, M. S. and H. C. Longuet-Higgins, two of whom have later become university professors." Ibid., p. 13.

many meanings of 'model.' Furthermore, there is controversy as to what meaning or meanings are adequate. Consider that some would use 'theory' and 'model' interchangeably. H. Kacser states as much:

> . . . all models, theories, laws, and hypotheses have the same logical status . . .[6]

Herbert Simon assumes as much. He wrote a series of essays in which he

> tried to set forth a consistent body of theory of the rational and nonrational aspects of human behavior in a social setting.[7]

Then he published them under the title, 'Models of Man.'[8] Others would warn against such identification, and even assert that it is unnecessary.

> The first danger is that theory will be identified with a model for it, so that the objects with which the model is concerned . . . will be supposed actually to be the same as the theoretical concepts of the theory. . . . Thinking of scientific theories by means of models is always as-if thinking; hydrogen atoms behave (in certain respects) as if they were solar systems each with an electronic planet revolving round a protonic sun. But hydrogen atoms are not solar systems; it is only useful to think of them as if they were such systems if one remembers all the time they are not.[9]

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[6] "Kinetic Models of Development and Heredity," in Models and Analogues in Biology, Symposia of the Society for Experimental Biology, Number XIV, Cambridge: The University Press, 1960, p. 15.

[7] Models of Man, New York: John Wiley & Sons, Inc., 1957, p. vii.

[8] Op. cit.

[9] R. B. Braithwaite, Scientific Explanation, Cambridge: University Press, 1953, p. 93.

> The first unnecessary use of "model" is a synonym for "theory."[10]

A second sorting is called for in regard to theory and model.

Brodbeck's analysis suggests that some theories are called 'models,' because they have certain characteristics; and she is quite right in asserting that these characteristics are found to hold for any theory.

> . . . uncertainty, selection, idealization, and quantification are characteristic to a greater or lesser degree of most worth-while theories.[11]

Furthermore, I agree with her rhetorical query:

> What then is gained, except unnecessary confusion, by calling theories which share in some or all of these characteristics "models"?[12].

Yet there is a sense in which 'theory' is synonymous with 'model.' There is another characteristic that furnishes the basis for equivalence of logical status.

The distinction between being a model of and being a model for will furnish such a basis. To be a model of is to represent something, and to be a model for is to be represented in something. A common sense context will help to clarify what I have in mind. Consider a young woman who is hired by the Powers' Agency as a model.

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[10]M. Brodbeck, "Models, Meaning, and Theories," in Symposium on Sociological Theory, ed. L. Gross, Illinois: Row Peterson and Company, 1959, p. 381.

[11]Ibid., p. 383.

[12]Ibid.

The assumption is that the reason for her employment is her representation of the ideal in figure and manner. She is a model of the ideal. The ideal in this case might be a tendency toward the real, a statistical norm, although cursory observation on my part would indicate not. She in turn becomes a model for other young women. Others attempt to be representations of her figure and manner. I shall designate a model which is a representation, 'a first-order model;' and a model which is being represented, 'a second-order model.'[13] Returning to the illustration, the young woman is first a model of the ideal (first-order model), and is then a model for other young women (second-order model). First she represents, then she is represented.

Before bringing the distinction between first-order and second-order models to bear on the concept of theory, this concept must be disinvolved. A theory is a system of statements. The statements are conjectures that characterize objects, and so are called 'hypotheses.' When the hypotheses are evaluated and found to be adequate in terms of instances, they come to be called 'laws.' For example, Charles' conjecture or hypothesis which characterizes gases as follows:

> if the temperature of a gas increases, its volume will increase provided the pressure is constant, and the pressure will increase if the volume is constant

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[13]The terms 'first-order model' and 'second-order model' were suggested to me by W. T. Williams's paper, "The Problem of Communication in Biological Teaching," in Models and Analogues in Biology, op. cit. pp. 243-249.

has been evaluated and found to be adequate in terms of instances. It is called 'Charles' law.' The statements of a theory form a system, in so far as they are related so that from some as premises all the others follow. To return to our illustration, Charles' law is a statement within a system of statements known as 'the Kinetic Theory.' It is related to other statements, such as

> a gas is composed of perfectly elastic properties moving at a high velocity.

These statements are premises from which Charles' law may be deduced.

The question as to the logical equivalence of 'theory' and 'model' is now answerable. All theories represent, because the statements of which they consist are characterizations of objects. They are models of objects. In the words of Wittgenstein:

> 4.01 Der Satz ist ein Bild der Wirklichkeit.
>
> Der Satz ist ein Modell der Wirklichkeit, so wie wir sie uns denken.[14]

Thus, 'theory' and 'model' are logically equivalent terms, provided 'model' is used in a first-order sense. I propose to use the adjective, 'representational,' to designate such a sense. 'Theory' and 'representational model' are synonymous terms.[15] 'Kinetic Theory'

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[14]Tractatus Logico-Philosophicus, London: Kegan Paul, Trench, Trubner & Co., 1922, p. 62.

[15]The relationship of logical equivalence or synonymity holds when the universe of discourse is science. Consider a universe of discourse which also includes first-order models, such as the young woman hired by the Powers' Agency. In this case, there would be first-order models that were not theories, yet all theories would be first-order models. The relationship would be that of class inclusion, where the class of theories is included in the class of first-order models ($T \subset M$, not $T \equiv M$).

and 'Kinetic Representational Model' are synonyms.

The separation of models into first-order and second-order makes patent that models relating to retroduction or the devising of theory must be second-order. What is wanted is a point of view which can be represented. This process of representing results in the conjecture or theory. Furthermore, it is not the case that the conjecture or theory is logically equivalent to the point of view. The point of view is a model for the theory, and is not the same as the theory. It is represented in the theory.

Since the way of inquiring that would result in science without defect rests upon the devising of theory, retroduction, one only can deplore current usage of the term 'model' which does not take into account the second-order sense. One would have no major objection to the following remarks of Paul Meadows:

1st order model

> The formulation of a Representational model consists in conceptually marking off a perceptual complex. . . . Every model is a pattern of symbols, rules, and processes regarded as matching, in part or in totality, an existing perceptual complex. Each model stipulates, thus, some correspondence with reality, some relevance of items in the model to the reality, and some verifiability between model and reality.[16]

if he had modified the term, 'model,' with the term, 'representational.' Could it not be the case that 'physics' could be substituted for 'psychology' in the statement to follow of Broad, if physicists had not paid close attention to second-order models?

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[16]"Models, Systems and Science" in American Sociological Review, Vol. 22, No. 1, 1957, p. 4.

> Poor dear Psychology, of course, has never got far beyond the stage of medieval physics, except in its statistical developments, where the labours of the mathematicians have enabled it to spin out the correlation of trivialities into endless refinements.[17]

But physicists did; and among them was the outstanding nineteenth century theoretician, James Clerk Maxwell, who wrote concerning the use of a point of view to devise conjecture or theory. To elucidate this process and the relationship between the point of view and the theory, one essay of Maxwell's--"On Faraday's Lines of Force"--is of particular significance. First, he spoke of the relationship in terms of physical analogy:

> In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies. By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other.[18]

Then he cited an example:

> The laws of the conduction of heat in uniform media appear at first sight among the most different in their physical relations from those relating to attractions. The quantities which enter into them are temperature, flow of heat, conductivity. The word force is foreign to the subject. Yet we find that the mathematical laws of the uniform motion of heat in homogeneous media are identical in form with those of attractions varying inversely as the square of the distance. We have only to substitute source of heat for centre of attraction, flow of heat

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[17]C. D. Broad, 'The "Nature" of a Continuant,' in Readings in Philosophical Analysis, ed. by Feigl and Sellars, New York: Appleton-Century-Crofts, Inc., 1949, p. 476. Since this is reprinted from Examination of McTaggart's Philosophy which was published in 1933, one wonders whether a statement made today by Broad would be so condemnatory. I should think not.

[18]The Scientific Papers of James Clerk Maxwell, ed. by W. D. Niven, Cambridge: The University Press, 1890, Vol. 1, p. 156.

> for *accelerating effect of attraction* at any point, and *temperature* for *potential*, and the solution of a problem in attractions is transformed into that of a problem in heat.
> This analogy between the formula of heat and attraction was, I believe, first pointed out by Professor William Thomson in the Camb. Math. Journal, Vol. III.[19]

Finally, he set forth the point of view which he used to devise his theory of electricity:

> It is by use of analogies of this kind that I have attempted to bring before the mind in a convenient and manageable form, those mathematical ideas which are necessary to the study of the phenomena of electricity. The methods are generally those suggested by the processes of reasoning which are found in the researches of Faraday . . .[20]

> By referring everything to the purely geometrical idea of the motion of an imaginary fluid, I hope to attain generality and precision . . . If the results of mere speculation which I have collected are found to be of use to experimental philosophers, in arranging and interpreting their results, they will have served their purpose . . .[21]

Surely the passage of time since Maxwell's day has indicated that the generality and precision (theory) achieved through the idea of the motion of an imaginary fluid (point of view) did achieve arrangement and interpretation (integration) of electrical phenomena. The purpose was served.

Maxwell's discussion has clarified the nature of second-order models as they function in devising theory. The theory of mechanics furnished substance (concepts) and form or structure (ways of relating concepts) which were represented in another system of

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[19] *Ibid.*, p. 157.

[20] *Ibid.*

[21] *Ibid.*, p. 159.

statements. So the theory of electricity emerged. The theory of mechanics was a devising model for the theory of electricity. Yet it was not the case that the theory of electricity was equivalent to a part or all of the theory of mechanics. There was partial similarity in substance (isosubstantism[22]) and in form (isomorphism), nevertheless the theory of electricity contained more than the substance and form from the theory of mechanics which were represented in it.[23] This may be generalized as follows: Theory characterizing other objects might furnish concepts or ways of relating concepts or both which can be utilized to characterize the objects we are pondering. If it does, it is a model for devising theory. One theory serves as a point of view from which to set forth another theory. A more precise rendering of this generalization might be:

A theory X is a devising model for a theory Y when

1. Y contains substance or form or both, Z, from X,
2. whatever is true of Z is true of Y, but
3. not whatever is true of Y is true of X.

Condition 1 permits representation of X in Y through substance alone, form alone, or both. Condition 2 insures either isosubstantism, isomorphism, or both. Condition 3 stipulates that Y cannot be

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[22] I have coined this term, since there is none that I know of to indicate sameness in substance.

[23] Peirce used the term, 'retroduction,' to indicate that what is involved is a leading back. The theory or conjecture that emerges (conclusion) contains more than the theory or point of view from which it emerges (premises). The implication, then, can only hold from the conclusion to the premises.

equivalent to a part of or all of X.

Brodbeck treats of a special case, where the theory that emerges represents the form of the laws of the theory which is the devising model (Y contains Z where Z is the form of the laws from X), when she defines 'model.'

> Two theories whose laws have the same form are isomorphic or structurally similar to each other. If the laws of one theory have the same form as the laws of another theory, then one may be said to be a model for the other.[24]

Perhaps the tendency[25] to restrict the devising model to this special case is explained by the ease with which the operations involved can be specified. Consider once more Maxwell's example of a physical analogy.[26] Consider also what Brodbeck has to say.

> Suppose that one area, as indicated by a set of descriptive concepts, for which a relatively well-developed theory is at hand is said to be a model for another area, about which little is as yet known. The descriptive terms in the theory of the better-known area are put into one-to-one correspondence with those of the "new" area. By means of this one-to-one correspondence, the laws of one area are "translated" into laws of the other area. The concepts of the better-known theory are replaced in the laws by the concepts of the new area. This replacement results in a set of laws or hypotheses about the variables of the new area. If observation shows these hypotheses to be true, then the laws of both areas have the same form. The lawful connections are preserved and the two theories are completely isomorphic to each other. For example, suppose it is wondered whether rumors spread like diseases. That is, can the laws of epidemiology, about which quite a bit is known, be a model for a theory of rumor transmission? Or,

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[24] *Op. cit.*, p. 379.

[25] Brodbeck overcomes this tendency when she speaks of models in physics. *Ibid.*, pp. 398-401.

[26] See pages 8 and 9 of this paper.

> to say the same thing differently, do the laws about rumors have the same form as the laws about diseases? The descriptive concepts in the laws of epidemiology are first of all replaced by letter variables. This reveals the form of the laws. The concepts referring to diseases are put into one-to-one correspondence with those referring to rumors. The letter variables in the epidemiological laws are replaced by the descriptive terms referring to rumors. This results in a set of hypotheses about rumors, which may or may not be confirmed. If, optimistically, these laws are confirmed, then the two theories have the same form.[27]

Another special case is the mathematical model. This case is assuming more and more importance in all realms.[28,29] However, there is a great deal of confusion both as to the nature and role of the mathematical model.

Confusion arises as to the nature of the mathematical model, because sometimes it is considered as a logical model and sometimes it is not.[30,31] I propose following Nagel's lead and not bothering to sort out mathematical models from logical ones. The category including both would be the formal model.[32] This makes sense for

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[27] Op. cit., p. 379.

[28] Mathematical Thinking in the Social Sciences, ed. by Paul F. Lazarsfeld, Illinois: The Free Press, 1954.

[29] J. G. Kemeny and J. L. Snell, Mathematical Models in the Social Sciences, Boston: Ginn and Co., 1961.

[30] Kenneth J. Arrow, "Mathematical Models in the Social Sciences" in The Policy Sciences, ed. by Lerner and Lasswell, Stanford: University Press, 1951, pp. 129-154.

[31] F. H. George, "Models and Theories in Social Psychology" in Symposium on Sociological Theory, op. cit., pp. 311-347.

[32] Ernest Nagel in The Structure of Science, New York: Harcourt, Brace and World, Inc., 1961, pp. 110-111 distinguishes formal and substantive analogies.

various reasons. It is no easy task to disentangle mathematics from logic. Some mathematics can be stated in terms of logic. Branches of modern mathematics can no longer be characterized as involving numerical considerations or space considerations. Yet both mathematics and logic are formal in nature. They both deal with nothing other than ways of relating. The statement from the sentential calculus (a logical calculus), '$p \supset q$,' tells us that two different statements, p and q, may be related so that it is not the case that both p is true and q is false. The mathematical statement $ab = 1$ indicates that two quantities, a and b, vary inversely with each other. If we generalize the term 'theory' to mean formal theories as well as scientific ones,[33] then the generalization regarding the devising model[34] could hold for formal models. They would be a special case where Z is form.

The role of the formal model is not clear, because often it is not recognized that the same formal model can be represented in many different scientific theories. Of course, it is obvious that all scientific theories must include necessarily a way or ways of relating concepts. The sentential calculus is an example of a formal model that is used in many contexts. As an illustration, $c \equiv . a \cdot b$ may be used to structure the following statements:

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[33]In the prior portion of the text, I have used 'theory' only in the sense of scientific theory.

[34]See page 10 of this paper.

the level of interaction in a group is greater

the amount of friendliness in a group is greater

the amount of activity carried on by members in a group is greater

by allowing 'c,' 'a,' and 'b' to denote each in turn. Thus, the level of interaction in a group will be greater if and only if both the amount of friendliness and the amount of activity carried on are greater. Also $c \equiv . a \cdot b$ may be used to structure a firing relationship between two excitatory input fibers, a and b, and one output fiber, c.

Yet another confusion arises as to the nature of the formal model due to the proclivity of some to draw. The formal does not become the non-formal when it is schematized. It may, however, become less precise. The above firing relationship is pictured as:

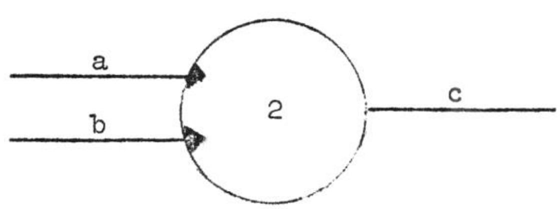

Figure 1[35]

A more familiar illustration might be the schematizing of class

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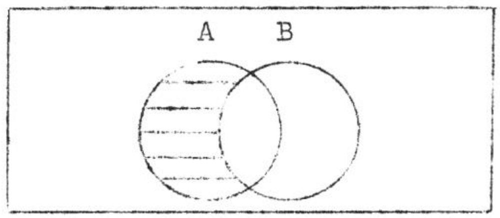
[35]F. H. George, op. cit., p. 316.

inclusion, $A \subset B$, in a Venn diagram:



A final confusion relates to the role of the formal model. The formal model can devise, but it can also explicate and evaluate. To explicate is not to contribute theory, but is to clarify and complete theory. Such an explicatory formal model is exemplified by linear graph theory as Cartwright and Harary use it to clarify and complete Heider's theory.[36] To evaluate is not to contribute theory, but is to determine the objects falling within the range of the characterization or theory. It is a statistical argument, and

> . . . in no other branch of science is the word model as often and as consciously used as in statistics.[37]

One has but to mention the normal curve for illustrative purposes.

The context of models marked off by Hertz has been sorted

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[36]Dorwin Cartwright and Frank Harary, "Structural Balance: A Generalization of Heider's Theory" in The Psychological Review, Vol. 63, No. 5, 1956, pp. 277-293.

[37]Hans Freudenthal, "Models in Applied Probability," in The Concept and the Role of the Model in Mathematics and Natural and Social Sciences, ed. by Freudenthal, Holland: D. Reidel Publishing Co., 1961, p. 79.

about. Surely our child, retroduction, has been given meaning in terms of a devising model. What is necessary is the growth of the child, its use in the social sciences. Or as Feigl has put it

> It would seem that the time has come for this sort of "Anschluss."[38]

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[38] Herbert Feigl, "Principles and Problems of Theory Construction in Psychology," in Current Trends in Psychological Theory, Wayne Dennis *et al.*, Pittsburgh: The University Press, 1951, p. 206.